

The profit-sharing rule that maximizes sustainability of cartel agreements

João Correia-da-Silva ^{1,2}
Joana Pinho ³

¹ FEP-UP, School of Economics and Management, University of Porto

² CEF.UP, Research Center in Economics and Finance, University of Porto

³ Faculdade de Ciencias Económicas e Empresariais, University of Vigo

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João Correia-da-Silva*

CEF.UP and Faculdade de Economia. Universidade do Porto.

Joana Pinho[†]

Faculdade de Ciencias Económicas e Empresariais. Universidade de Vigo.

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Abstract. We propose a profit-sharing rule that maximizes sustainability of cartel agreements. This rule is such that the critical discount factor is the same for all the firms. If a cartel applies this rule, then asymmetries among firms may not hinder collusion (contrarily to the typical finding in the literature). In the simplest case of a Cournot duopoly in which firms differ in their stocks of capital, we find that the cartel is the least sustainable when one of the firms is approximately two times bigger than the other.

Keywords: Collusion sustainability, Profit-sharing rule.

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1 Introduction

The literature on the sustainability of cartels typically studies an infinitely repeated game where each of the firms may either: act in accordance with a collusive agreement that specifies each firm's output and the price charged to consumers with the objective of maximizing the profit of the cartel; or break the agreement, for example, by increasing its output and decreasing the price it charges, taking advantage of the fact that the other firms are restricting output and charging a monopoly price. If one of the firms breaks the agreement, the industry becomes competitive and thus profits in the subsequent periods are lower.

The choice of a firm between these two options depends on the relative importance that it gives to present and future profits. If the discount factor is sufficiently close to unity (meaning that the same importance is given to present and future profits), the firm will prefer to abide by the cartel rules; if it is sufficiently close to zero, then the firm will prefer to break the cartel to enjoy a higher profit in the present, in spite of suffering the consequent decrease of future profits. The threshold value of the discount factor, above which no firm finds it profitable to break the agreement, is an indicator of the sustainability of the cartel.

If firms are heterogeneous, sharing the profit of the cartel is not a trivial matter. The importance of avoiding side-payments in order to decrease the probability of detection yields a natural profit-sharing rule: allocate production efficiently and then let each firm keep its individual profit.¹ An equally natural profit-sharing rule is based on the solution of the Nash bargaining game: firms get the profits that they would obtain under competition plus an equal share of the increase of the industry profit generated by the cartel agreement.²

Depending on the type of firm heterogeneity and on the profit-sharing rule that is adopted, some firms will be more prone to break the cartel than others. Since the sustainability of the cartel only depends on the behavior of the firm that has the strongest propensity to deviate, it seems to be straightforward that firm heterogeneity decreases the sustainability of cartels. This conclusion is supported by the theoretical contributions of Harrington (1989), Compte *et al.* (2002), Kühn (2004), Vasconcelos (2005), Miklós-Thal (2011) and Brandão *et al.* (2012).

To address the issue of sustaining collusion in the presence of asymmetries among firms, we propose a profit-sharing rule (i.e., a system of side-payments) that maximizes the sustainability of the cartel. With this rule, all the firms end up having the same threshold discount factor. If one of the firms had a higher threshold factor (stronger propensity to break the cartel), then the sustainability of the cartel agreement could be increased by

¹This profit-sharing rule is the most commonly used in the literature. See the discussion by Bos and Harrington (2010).

²This alternative has been proposed by Osborne and Pitchik (1993).

marginally increasing this firm's share of the cartel profits.

Applying this profit-sharing rule to a simple example with two firms that differ in their stocks of capital (and, as a result, in their marginal costs of production), we find that the relationship between the asymmetry between firms and the sustainability of collusion is U-shaped. The cartel is the least sustainable when one of the firms is approximately two times bigger than the other. It is more sustainable when firms are symmetric, and much more sustainable when one of the firms has almost all the industry capital.

2 Model

Consider an industry composed by heterogeneous firms in a stationary infinite-horizon setting. The objective of each firm is to maximize the discounted value of its flow of profits: $\sum_{t=0}^{\infty} \delta^t \Pi_{it}$, where Π_{it} denotes the profit of firm i in period t . All firms have the same discount factor, $\delta \in (0, 1)$.

Before period $t = 0$, firms $i \in \{1, \dots, N\}$ establish a collusive agreement.³ This agreement lasts forever, as long as there are no defections. If any of the firms violates the agreement, firms engage in competition in all the subsequent periods (firms use grim trigger strategies). There is no renegotiation.

Let Π_i^s denote the single-period profit of firm i in each of the possible competitive scenarios: $s = c$ if firms are competing; $s = m$ if firms are colluding; and $s = d$ if firm i unilaterally deviates from the collusive agreement.

Firms prefer to abide by the collusive agreement if the discounted value of the flow of collusive profits exceeds the sum of the deviation profit with the discounted value of the flow of the subsequent competitive profits. More precisely, collusion is sustainable if and only if the following incentive compatibility constraint is satisfied for all firms $i \in \{1, \dots, N\}$:

$$\sum_{t=0}^{+\infty} \delta^t \Pi_i^m \geq \Pi_i^d + \sum_{t=1}^{+\infty} \delta^t \Pi_i^c \Leftrightarrow \delta \geq \frac{\Pi_i^d - \Pi_i^m}{\Pi_i^d - \Pi_i^c} \equiv \delta_i^*. \quad (1)$$

Therefore, the critical discount factor for collusion sustainability is: $\delta^* = \max_{i \in \{1, \dots, N\}} \delta_i^*$.

The magnitude of each firm's critical discount factor depends on how the profit of the cartel is shared among firms. Denote by $\lambda_i \in (0, 1)$ the profit-share of firm i : $\Pi_i^m = \lambda_i M$, where M denotes the industry profit under collusion.

³We take the collusive agreement as a given. It may include or not all the firms in the industry, may maximize or not the profits of the cartel, and may stipulate the output and price set by each firm, a geographical sharing of the market, or any other type of collusive behavior.

We propose a profit-sharing rule that maximizes sustainability of collusion. Firms choose the allocation, $\lambda \equiv (\lambda_1, \dots, \lambda_N)$, that minimizes δ^* .

Proposition 1. *The allocation of the industry profit, λ^* , that maximizes sustainability of collusion is such that:*

$$\delta^* = \frac{D - M}{D - C} \quad \text{and} \quad \lambda_i^* = \frac{M - C}{M(D - C)} \Pi_i^d + \frac{D - M}{M(D - C)} \Pi_i^c, \quad (2)$$

where $C = \sum_{i=1}^N \Pi_i^c$ is the industry profit under competition and $D = \sum_{i=1}^N \Pi_i^d$ is the sum of the deviation profits of the firms.

Proof. It is rather trivial to show that $\delta_i^* = \delta_j^*, \forall i, j$. Let $A = \{i : \delta_i^* = \delta^*\}$ and $B = \{i : \delta_i^* < \delta^*\}$. If B were not empty, then a marginal increase of the profit shares of the firms in A at the cost of the profit shares of the firms in B would decrease δ^* .

The N incentive compatibility constraints (1) are satisfied with the equality sign. We have, therefore, a linear system of $N + 1$ equations with $N + 1$ unknowns:

$$\begin{cases} \delta^* = \frac{\Pi_i^d - \lambda_i^* M}{\Pi_i^d - \Pi_i^c}, \quad \forall i \\ \sum_{i=1}^N \lambda_i^* = 1, \end{cases}$$

whose solution is (2). □

Observe that the resulting critical discount factor is always lower than 1.

3 Cournot duopoly with asymmetric production costs

Consider the case of an infinitely-repeated Cournot duopoly with homogeneous goods but asymmetric production costs. In each period $t \in \{0, 1, 2, \dots\}$, firms 1 and 2 simultaneously choose their output levels, q_{1t} and q_{2t} . The price charged to consumers is the same for both firms and given by:

$$P_t = 1 - (q_{1t} + q_{2t}).$$

Firms need capital to produce. A firm with more capital can produce the same units of output at a lower cost. The cost function of firm i is given by:⁴

$$C_i(q_i) = \frac{q_i^2}{2k_i},$$

where k_i is the capital stock of firm i . For simplicity, let $k_1 = k$ and $k_2 = 1 - k$.

⁴This is a simplified version of the cost function suggested by Perry and Porter (1985).

Under competition, each firm produces the quantity that maximizes its individual profit:

$$\max_{q_i} \left\{ (1 - q_i - q_j) q_i - \frac{q_i^2}{2k_i} \right\}.$$

The competitive output and profit of each firm $i \in \{1, 2\}$ is given by:

$$q_i^c = \frac{k_i}{(1 + k_i)(1 + S)} \quad \text{and} \quad \Pi_i^c = \frac{k_i(1 + 2k_i)}{2(1 + k_i)^2(1 + S)^2},$$

where $S = \sum_{i=1}^2 \frac{k_i}{1 + k_i}$.

We follow most theoretical contributions that study collusion sustainability in assuming that, along the collusive path, firms maximize their joint profit.

$$\max_{(q_1, q_2)} \left\{ (1 - q_1 - q_2) (q_1 + q_2) - \frac{q_1^2}{2k} - \frac{q_2^2}{2(1 - k)} \right\}.$$

Under perfect collusion, the output of each firm is proportional to its capital stock, $q_i^m = \frac{k_i}{3}$, and the profit of the industry is $M = \frac{1}{6}$.

If firm i decides to disrupt the collusive agreement, it will produce the quantity that maximizes its individual profit, assuming that the other firm produces the collusive quantity:

$$\max_{q_i} \left\{ \left(1 - q_i - \frac{k_j}{3} \right) q_i - \frac{q_i^2}{2k_i} \right\}.$$

The deviation output and the corresponding single-period profit are:

$$q_i^d = \frac{k_i(2 + k_i)}{3(1 + 2k_i)} \quad \text{and} \quad \Pi_i^d = \frac{k_i(2 + k_i)^2}{18(1 + 2k_i)}.$$

Using (2), we obtain the critical discount factor that is associated with the profit-sharing rule that maximizes collusion sustainability (illustrated in Figure 1):

$$\delta^* = \frac{(3 - 4k + 4k^2)(1 + k - k^2)^2}{6 + 3k - 12k^2 + 14k^3 + 3k^4 - 12k^5 + 4k^6}. \quad (3)$$

Collusion is easiest to sustain if one firm owns almost the whole industry capital (i.e., when $k \rightarrow 0$ or $k \rightarrow 1$). On the contrary, collusion is less likely when one firm owns approximately 33.2% of the industry capital. The most surprising result is that symmetry between firms does not necessarily facilitates collusion. This is in contrast with the existing literature, which defends symmetry across firms as facilitating collusion (Motta, 2004).

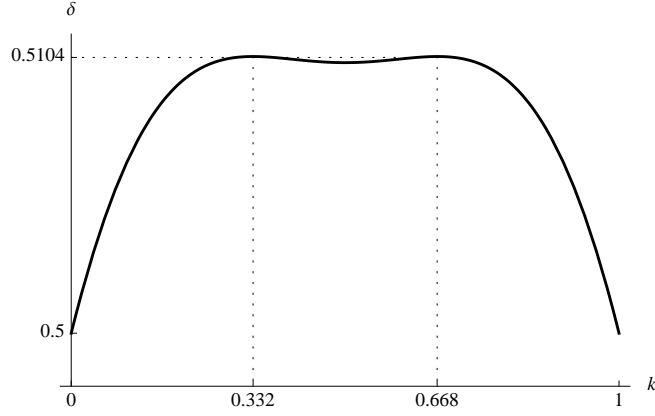


Figure 1: Critical discount factor of both firms with the profit-sharing rule that maximizes collusion sustainability.

If firms divide their joint profit according to their relative bargaining power, the share of firm i is the solution of the associated Nash bargaining game:

$$\max_{\lambda_i^N} \{ (\lambda_i^N M - \Pi_i^c) [(1 - \lambda_i^N) M - \Pi_j^c] \}.$$

After some algebra, we obtain the critical discount factors for the firms to abide by the collusive agreement (illustrated in Figure 2):

$$\delta_1^N = \frac{-4 - 8k + 45k^2 + 62k^3 - 68k^4 + 6k^5 + 51k^6 - 57k^7 + 6k^8 + 6k^9}{3k^2(-1 - k + k^2)(3 - 2k + 2k^2)(-4 - 6k + 3k^2 + k^3)}$$

and

$$\delta_2^N = \frac{-39 + 35k + 165k^2 - 285k^3 + 92k^4 + 207k^5 - 324k^6 + 207k^7 - 60k^8 + 6k^9}{3(-1 + k)^2(-1 - k + k^2)(3 - 2k + 2k^2)(6 + 3k - 6k^2 + k^3)}.$$

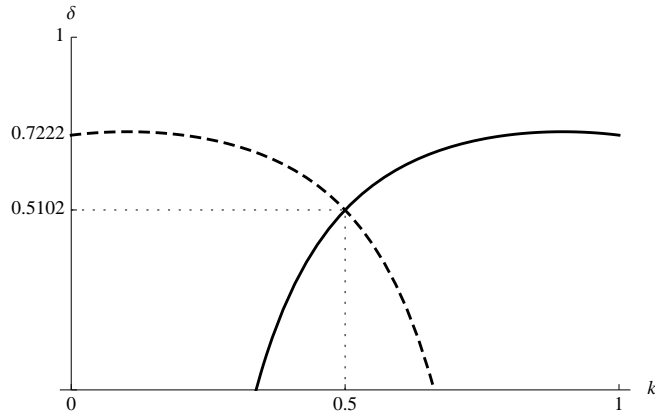


Figure 2: Critical discount factors of firm 1 (solid line) and firm 2 (dashed line) if profits are shared according to the Nash bargaining rule.

We conclude that the smaller firm is the most tempted to deviate.

If side-payments between firms are not feasible, the critical discount factors (above which firms do not disrupt the collusive agreement) are:

$$\delta_1^P = \frac{(1-k)(1+k-k^2)^2}{k^2(4+6k-3k^2-k^3)} \quad \text{and} \quad \delta_2^P = \frac{k(1+k-k^2)^2}{(1-k)^2(6+3k-6k^2+k^3)}.$$

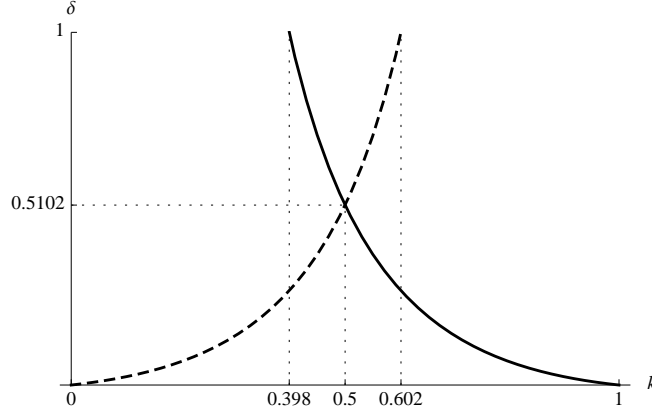


Figure 3: Critical discount factors for firm 1 (solid line) and firm 2 (dashed line), if there are no side-payments.

In this case, it is the bigger firm that has the most incentives to deviate (see Figure 3). If $k \in (0, 0.39) \cup (0.61, 1)$, perfect collusion is not sustainable if there are no side-payments.

Figure 4 shows that the division of the industry profit along the collusive path that we propose significantly decreases the critical discount factor:

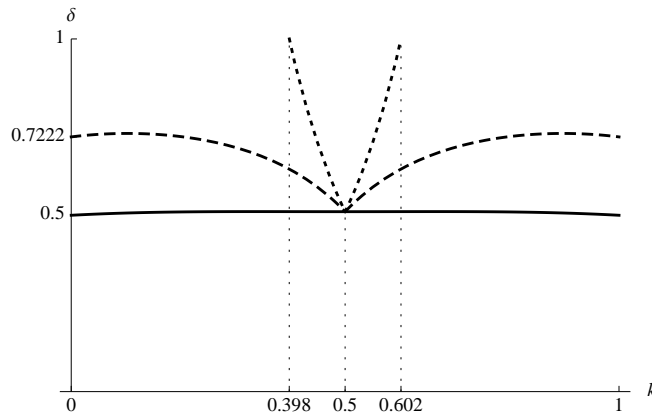


Figure 4: Comparison of the critical discount factors associated with the three profit-sharing rules: sustainability-maximizing (solid line), bargaining solution (dashed line), no side-payments (dotted line).

4 Conclusions

When the asymmetries between firms are very pronounced, perfect collusion without side-payments may not be possible. However, if firms share their collusive profits in an appropriate way, they become able to sustain perfect collusion (as long as the discount factor is high enough). With the profit-sharing rule that maximizes collusion sustainability, i.e., minimizes the critical discount factor, an asymmetry between firms may even increase collusion sustainability. This contrasts with the typical message in the literature.

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